J. agric. Engng Res. (1996) **63,** 205–218

A Geometrical Analysis of Inclined and Tilted Spherical Plough Discs

M. J. O'Dogherty;* R. J. Godwin;* M. J. Hann;* A. A. Al-Ghazal†

* Silsoe College, Cranfield University, Silsoe, Bedford MK45 2DT, UK
† King Faisal University, Al-Hassa, Saudi Arabia

(Received 27 February 1995; accepted in revised form 6 October 1995)

An analysis is presented of the effect of disc geometry in relation to its areas of contact with soil at the working depth, treating the disc as a segment of a thin spherical shell. In addition, the effects of the disc angle of tilt, the disc angle of inclination to its direction of motion and the working depth are examined in detail. Shallow and deep disc concavities are considered. A presentation is made of formulae derived for disc critical angle and depth and for bearing and pressure areas of contact with the soil on vertical and horizontal planes. In addition, a study was made of the overlapping of soil working areas when adjacent discs are working in a gang arrangement. The effects of inside and outside sharpening of the circumferential edge of the disc are also examined.

For the practical range of tilt angle (15° to 25°) and disc angle (35° to 55°) it is shown that the bearing area of the rear spherical area of discs is zero, so there is no soil contact with the rear surface of the disc. The vertical pressure area is only slightly affected by tilt angle and there is little difference for the two disc concavities. Disc angle and working depth have significant effects on this area. The horizontal presssure area is not affected by disc angle over its practical range. It has larger values for the 81 mm concavity than for the 51 mm concavity and is significantly influenced by tilt angle and working depth. Discs working in a gang, have overlapping of the areas of soil cut for disc angles and spacings (180 to 300 mm) adopted in practice. The area cut by an individual disc is not markedly affected by disc angle. Inside and outside sharpening does not significantly affect the overall findings concerning critical disc parameters. The flat bevelled surface formed by outside sharpening will generally be in contact with soil over the practical range of tilt and disc angles.

Notation

R_0	radius of rear spherical surface of disc
R_i	radius of front spherical surface of disc
t	thickness of disc
c	concavity of disc: depth from circumferential plane to centre of front spherical surface
r	radius of sharpened edge of disc circle in its circumferential plane
α	tilt angle of disc circumferential plane to vertical plane
β	angle of disc circumferential plane to direction of motion
d	depth of disc working from soil surface
g	spacing between disc centres in a disc gang
β_c	critical disc angle: the disc angle at which the rear spherical surface is no longer in contact with soil
L	half length of chord of intersection of circumferential circle with plane of soil surface
α_c	critical tilt angle: the tilt angle at which the rear spherical surface of the disc contacts the soil
R_s	radius of circle of intersection of soil surface with rear spherical surface of disc
d_c	critical depth: the depth below the soil surface at which the disc angle is equal to the critical angle
ϵ	angle between disc axis and a line from centre of spherical shell to edge of circumferential circle of disc plane
τ	half angle between radii of circumferential circle drawn to ends of its chord of intersection with soil surface

0021-8634/96/030205 + 13 $18.00/0

A_p vertical pressure area: area of interface of soil with the front spherical surface of the disc below soil level, projected on to a vertical plane normal to direction of motion

A_b vertical bearing area: area of interface of soil with the rear convex surface of the disc below soil level, projected on to a vertical plane normal to direction of motion for disc angles greater than the critical angle (β_c)

A_{ph} horizontal pressure area: area of contact of soil with the front spherical surface of the disc, projected on to a horizontal plane

A_c area of contact of soil with the front spherical surface of the disc, below the chord of intersection with the circumferential edge, projected on to a horizontal plane

A_{bh} horizontal bearing area: area of contact of soil with the rear spherical surface of the disc, projected on to a horizontal plane

A_{ov} area by which a disc overlaps the area of soil cut by a following disc, when discs are working in a gang, projected on to the vertical plane normal to the direction of motion

A_g area of soil cut by an individual disc, when discs are working in a gang, projected on to the vertical plane normal to the direction of motion

A_u Area of cross-section in a vertical plane of uncut soil between adjacent discs working in a gang

β_0 critical disc angle at which overlap occurs between the cut areas of soil of discs working in a gang for a given spacing

g_c critical disc spacing at which overlap occurs between the cut areas of soil of discs working in a gang for a given disc angle

δ height to cusp from working depth of the uncut area of soil cross-section in a vertical plane between adjacent discs working in a gang

x depth of an elemental strip of the vertical bearing area below the soil surface

h width of an elemental strip of the vertical bearing area at a depth, x, below the soil surface

R_s' radius of circle of intersection of soil surface with rear spherical surface of disc at a depth, x, below the soil surface

β' critical angle at a depth, x, below soil surface

r_1 radius of rear spherical surface of disc circle in its circumferential plane for an externally sharpened disc

d_1 effective depth of working of rear spherical surface of disc for an externally sharpened disc

ϵ_1, ϵ_2 modified values of angle, ϵ, for an externally sharpened disc

ψ bevel angle, defined as the angle between the bevelled surface and a line normal to the disc circumferential plane in the diametral plane of the spherical surfaces

ψ_s angle made by the bevelled flat surface of an externally sharpened disc in a horizontal plane, at soil level, to the direction of motion of the disc

w width of the bevelled edge of an externally sharpened disc in a diametral plane of the spherical surfaces

θ angle made by the bevelled flat surface of an externally sharpened disc to the tangent to the inner spherical surface of the disc in a diametral plane

Other minor terms are defined within the text

1. Introduction

The analysis presented in this paper is concerned with the geometry of plough discs which is relevant to the soil forces acting upon them when in work. Although some geometrical analyses and measurements have been conducted by investigators concerned with disc forces,[1–6] in general, they have not been fundamental. The most rigorous mathematical analysis was carried out by Abo El Ees[7] and Abo El Ees and Wills[8] but their analysis was restricted to discs for which the circumferential plane of the disc was vertical. The work presented in this paper is concerned with the general case of discs which are both inclined to the direction of motion of the plough and tilted at an angle to the vertical plane.[9] The object of this work was to examine the critical parameters of the disc and their effects on the soil engaging areas at the front and rear surfaces of the disc, in terms of their projected areas on the horizontal and vertical planes. Tillage forces are not considered in this paper.

2. Disc parameters and nomenclature

The cultivating disc is a segment of a thin spherical shell and its critical dimensions are shown in *Figs 1* to *5*. The principal parameters are the radius of the rear spherical surface, R_0 (*Fig. 1*) the thickness of the disc material, *t,* and the radius of the circle of the sharpened edge in the circumferential plane of the disc, *r* (*Figs 1* and *5*). Other disc parameters can be deduced in terms of these dimensions. The tilt angle of the disc, α (*Figs 1* and *5*) is defined as the angle which the circumferential plane makes with the vertical plane. The disc angle, β (*Fig. 2*) is the angle which the circumferential plane makes with the horizontal direction of motion of the plough. The depth of disc soil working, *d* (*Fig. 1*) is the vertical distance from the soil surface to the lowest point of the circle which forms the circumferential edge of the disc (*Fig. 1*).

For any disc, the angles α and β and the working depth, *d,* are the factors which determine the areas of the front and rear surfaces of the disc which engage the soil and are of importance in relation to the magnitudes of the consequent forces acting on the disc.

When a disc is working in soil the rear spherical surface will be in contact with soil if the disc angle is less than a critical angle, β_c (*Fig. 1*). This angle is reached when the direction of motion is tangential to the circle of intersection of the soil surface with the rear surface of the disc, at the point where it intersects the chord of intersection. If the disc angle, β, is equal to or greater than this angle, there is no soil contact with the rear surface and the vertical and horizontal bearing areas are zero. Expressions for the calculation of β_c are given in Section 3.1.

For a particular disc angle, β, there is a critical depth, d_c, for which the angle β is equal to the critical angle, up to which there is no soil contact with the

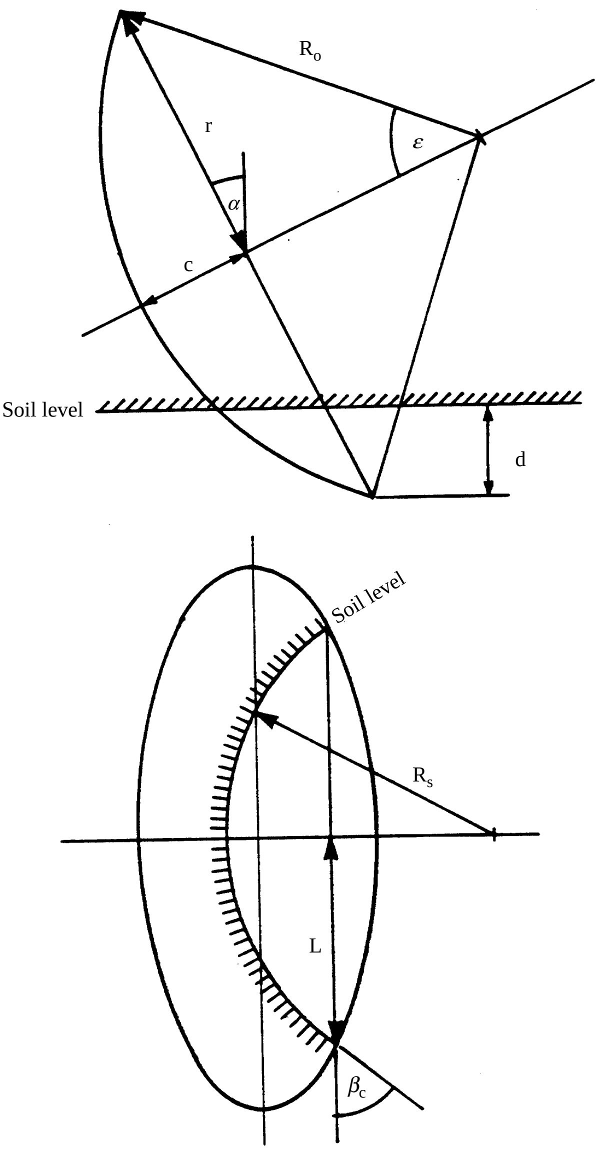

Fig. 1. Geometrical parameters of a disc tilted at an angle α to the vertical plane (upper: elevation; lower: plan)

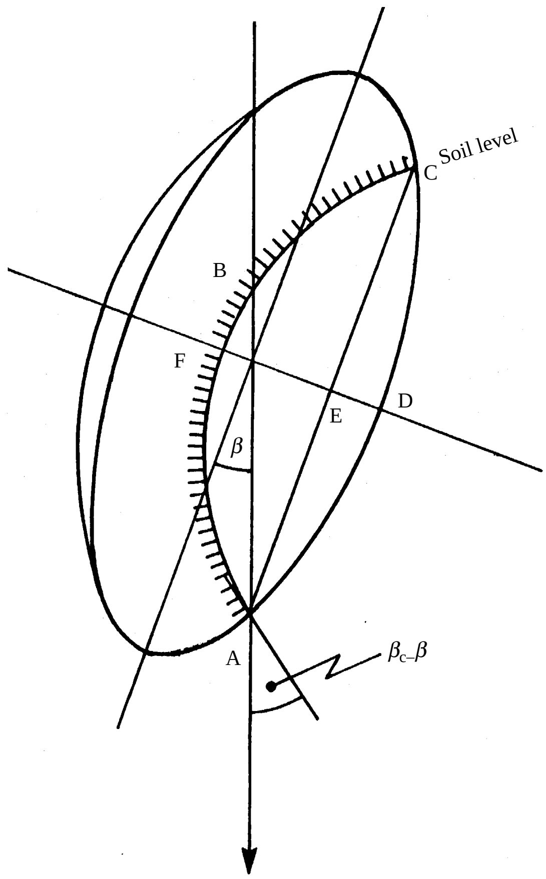

Fig. 2. Bearing and pressure areas of a disc at an angle β to the direction of motion projected on to a horizontal plane (A_{bh} = AFB; A'_{ph} = AFBC; A_c = AECD)

rear surface of the disc. An equation for determining values of d_c is given in Section 3.2.

The bearing area is defined as the area of the rear convex surface of the disc engaged with soil and will exist when $\beta < \beta_c$. Expressions for this area when projected on to a horizontal plane (A_{bh}) (area AFB in *Fig. 2*) are given in Section 4.2.2 and on to a vertical plane, normal to the direction of motion (A_b) (area ABE in *Fig. 3*), in Section 4.2.1.

The pressure area is defined as the area of the concave front surface of the disc engaged with the soil, which represents the area of soil cut by the disc. Expressions for this area projected on to a horizontal plane (A_{ph}) (area AFBCD in *Fig. 2*) are given in Section 4.2.1 and on to a vertical plane, normal to the direction of motion (A_p) (area BCDE in *Fig. 3*), in Section 4.1.1.

When working in gangs there is generally some overlap of the areas of soil cut by adjacent discs. For a particular disc spacing there is a critical angle, β_0, above which there will be overlap of the cut areas. There will also be a critical spacing, g_c, for a particular disc angle, at less than which there will be partial overlap of the worked areas. Expressions for these critical values are given in Section 5.1. The area of overlap between discs (A_{ov}) (area BFG in *Fig. 4*) is derived in Section 5.1 and the uncut area (A_u) (area CEF in *Fig. 4*) in Section 5.3. The gang area (A_g) (area BDEF in *Fig. 4*), or the area cut by a single disc, is considered in Section 5.2.

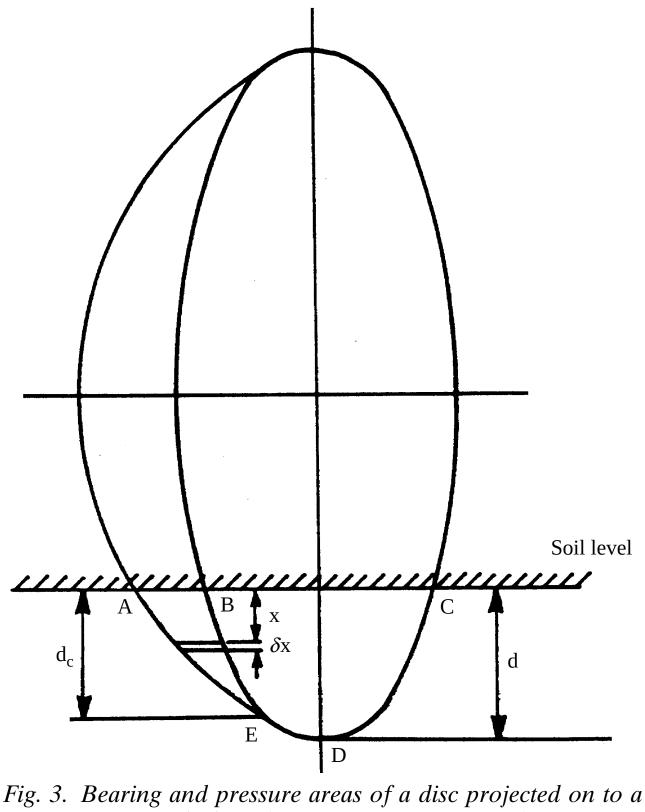

Fig. 3. Bearing and pressure areas of a disc projected on to a vertical plane ($A_b = ABE$; $A_p = BCDE$)

The disc may be sharpened externally or internally. The formulae derived in Sections 3 to 5 apply to an internally sharpened disc. The effect of external sharpening is discussed in Section 6 in relation to the width of the bevelled surface and to critical disc angle. Modifications required to the formulae for an externally sharpened disc are also discussed.

In practice, disc sizes are given in terms of the diameter of the circumferential or edge circle ($2r$) and their concavity (c), which is the depth from the plane of the edge circle to the inner spherical surface. Disc sizes range from 500 to 900 mm edge circle diameter with concavities over a range from 75 to 165 mm.[10,11] The thickness of discs is usually between 5 and 9 mm, depending on disc size. In terms of the basic parameters of the disc, the concavity, c, is given by $c = R_o - (R_o^2 - r^2)^{1/2} - t$ for inside sharpening and by $c = R_o - \{R_o - t)^2 - r^2\}^{1/2} - 2t$ for outside sharpening. In the calculations presented in this paper, a shallow and a deep disc are considered, sharpened internally and of dimensions representative of a typical disc size, which is usually in a range 600 to 700 mm edge circle diameter.[12]

For the shallow disc, $R_o = 915$ mm and $r = 315$ mm; for the deep disc $R_o = 560$ mm and $r = 305$ mm. The disc thickness (t) was equal to 5 mm in both cases. The discs were, therefore, of approximately equal size but of differing concavities. For the shallow disc $c =$ 51 mm and for the deep disc $c = 85$ mm, for inside sharpening.

In practice, for disc ploughs[12] the disc angle (β) ranges between 35° and 55° and is commonly in the range 40° to 45°. The tilt angle of the disc[12] (α) is normally in the range 15° to 25°. The spacing between discs (g) when working in a gang[12] is usually in the range 180 to 300 mm. In the calculations presented in this paper, the working depth of the discs[13] (d) was considered over a range from 60 to 180 mm.

3. Critical angles and depth

3.1. *Critical disc angle*

The critical disc angle, β_c, occurs when the disc angle, β, is such that the direction of motion is tangential to the rear spherical surface of the disc at the soil surface (*Figs 1* and *2*) and is given by

$$\sin \beta_c = L/R_s \tag{1}$$

In this expression L is half the length of the chord of

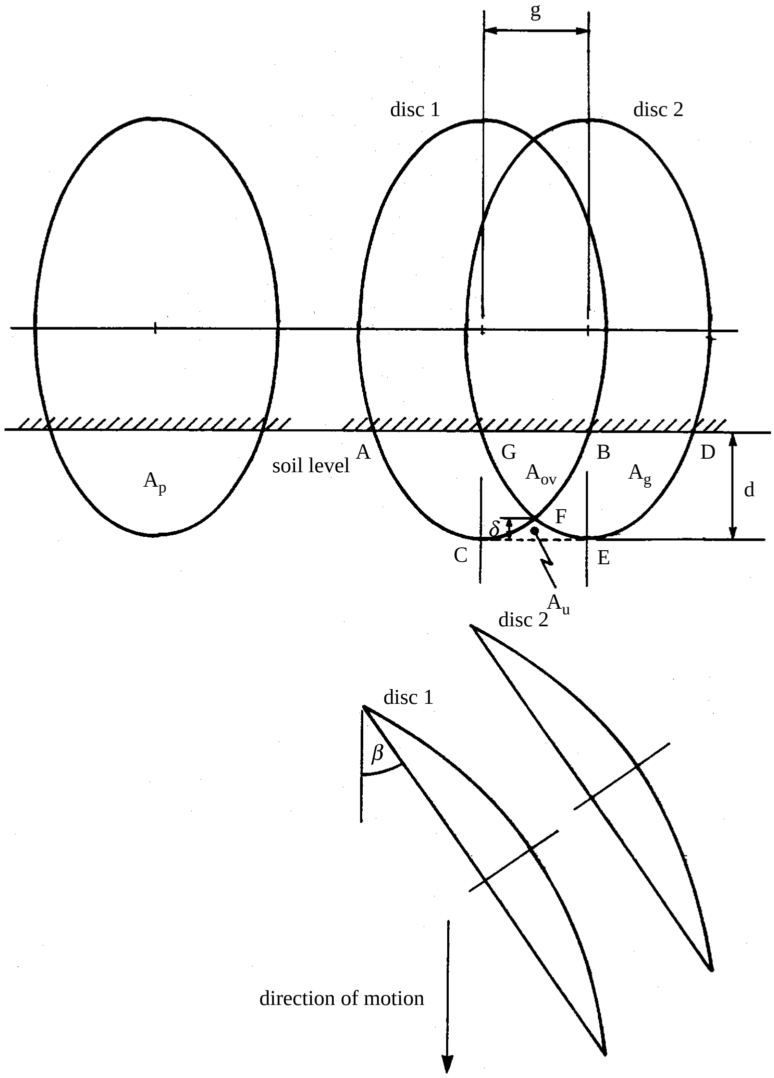

Fig. 4. Pressure areas and area of uncut soil of discs in a gang projected on to a vertical plane for $\alpha = 0°$ (upper: elevation; lower: plan; $A_p = ACFB = DEFG$; $A_{ov} = BFG$; $A_g = BFG$; $A_u = CFE$)

the disc edge circle in the plane of the soil surface (*Fig. 5*) and is given by

$$L = \{d(2r \cos \alpha - d)\}^{1/2} \sec \alpha \qquad (2)$$

The term, R_s, is the radius of the circle of intersection of the soil surface (*Fig. 1*) with the outer spherical surface of the disc and is given by

$$R_s = R_o[1 - \{\sin(\alpha + \epsilon) - d/R_o\}^2]^{1/2} \qquad (3)$$

where the angle ϵ (*Fig. 1*) is given by

$$\sin \epsilon = r/R_o$$

The variation of critical disc angle (β_c) with tilt angle (α) was calculated from Eqns (1) to (3) and is shown in *Fig. 6* for working depths of 60 and 180 mm, for the shallow and deep discs. In general, the effect of tilt angle was relatively small, particularly for the shallow discs. In practice, disc angle varies between 35° and 55° for plough discs and tilt angle between 15° and 20°, so that for typical settings ($\alpha = 20°$ and $\beta = 40°$ to 50°) the critical angle is exceeded for both discs. The only case where the critical angle is not exceeded is for the deep disc at tilt angles greater than 20° at a disc angle of 35°, when working at a depth of 180 mm.

3.2. *Critical tilt angle*

The critical tilt angle (α_c) occurs when the tilt of the disc is equal to the angle made by the tangent to the rear spherical surface, in a diametral plane of this surface, at a point where it intersects the circumferential plane of the disc. When this angle is reached the

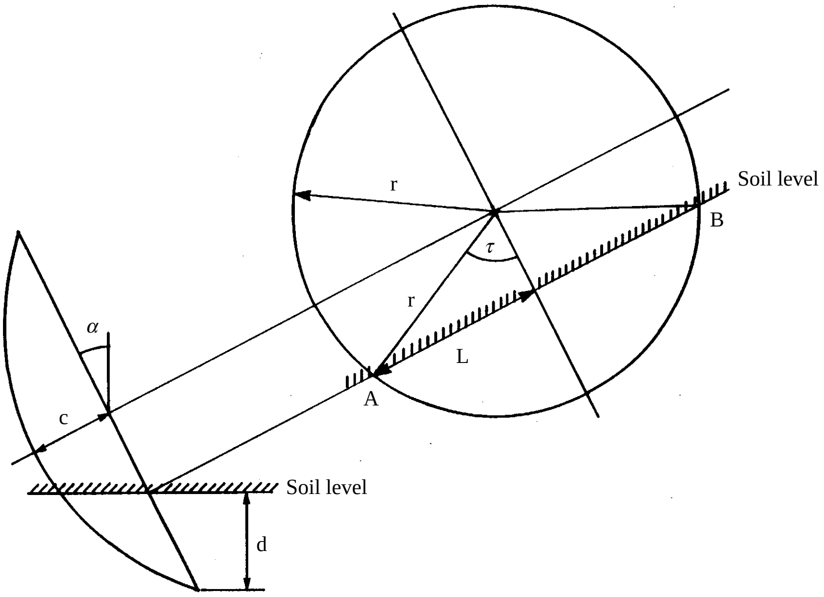

Fig. 5. Chord of intersection AB (=2L) of soil surface with disc circular edge for a disc inclined at an angle α to the vertical

rear surface will contact the soil. The value of the critical angle is given by

$$\alpha_c = \pi/2 - \epsilon \tag{4}$$

For the shallow disc, $\alpha_c = 69{\cdot}9°$ and for the deep disc $\alpha_c = 57°$, so that for the practical range of α (15° to 25°) the critical angle is not reached and the rear spherical surface of the discs will not contact the soil surface.

3.3. *Critical disc depth*

For a particular disc angle, β, there is a critical depth, d_c, for which β is the appropriate critical angle (*Fig. 3*). This angle can be determined from Eqns (1), (2) and (3) by obtaining an explicit expression for d_c for a given value of β. The equation is a quadratic one of the form

$$\mathrm{A}d_c^2 + \mathrm{B}d_c + \mathrm{C} = 0 \tag{5}$$

where $\mathrm{A} = 1 - \cos^2\alpha \sin^2\beta$; $\mathrm{B} = 2\cos\alpha\{R_o \cos\alpha \sin^2\beta \sin(\alpha+\epsilon) - r\}$ and $\mathrm{C} = R_o^2 \cos^2\alpha \sin^2\beta\{1 - \sin^2(\alpha+\epsilon)\}$

The positive solution of Eqn (5) is taken which gives a value of d_c which lies within the working depth of the disc.

Fig. 7 shows the effect of disc angle (β) on critical depth for tilt angles (α) of 10° and 30° for both the

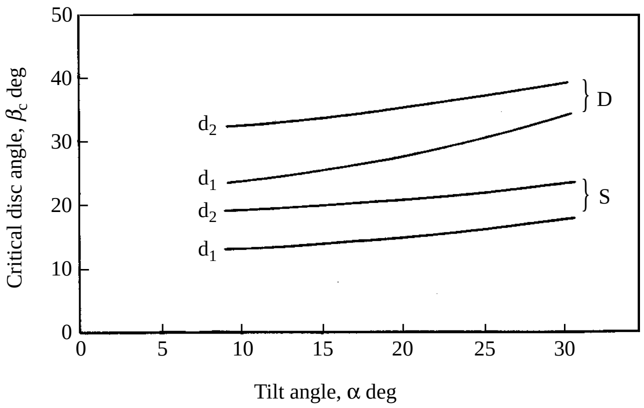

Fig. 6. Variation of critical disc angle (β_c) with tilt angle (α) for 85 mm disc concavity (D) and 51 mm disc concavity (S) (working depths: d_1 = 60 mm; d_2 = 180 mm)

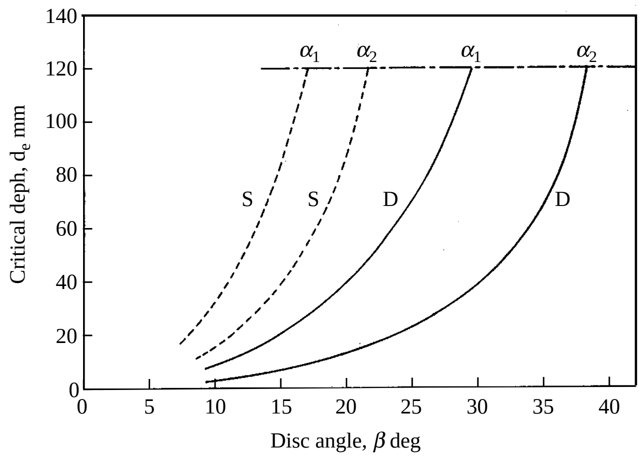

Fig. 7. Variation of critical depth (d_c) with disc angle (β) for 85 mm disc concavity (D) and 51 mm disc concavity (S) (tilt angles; α_1 = 10°; α_2 = 30°; curves intersect the working depth line at the critical angle)

shallow and deep discs. The critical depth increases rapidly from zero as disc angle increases until it is equal to the working depth of the disc, when the disc angle is equal to the critical angle. For a particular value of the disc angle, equal to or less than the critical angle, the critical depth shows a decrease with increasing tilt angle.

4. Projected areas of soil engagement with disc

4.1. *On a vertical plane normal to direction of motion*

4.1.1. *Vertical pressure area*

The area of interface between the soil and the disc inner concave spherical surface below the soil surface, when projected on to a vertical plane normal to the direction of motion, is designated the vertical pressure area, A_p (area BCDE in *Fig. 3*). It can be shown from the disc geometry that this area, normal to the direction of disc motion is given by

$$A_p = r^2(\tau - 0{\cdot}5 \sin 2\tau) \cos \alpha \sin \beta \qquad (6)$$

where $\sin \tau = L/r$ and L is given by Eqn (2). The angle τ is shown in *Fig. 5* and must be expressed in radians.

The effect of tilt angle (α) on the vertical pressure area is relatively small, with an increase of 5·7% over a range of α from 10° to 30°, at typical disc angle settings. There is also little difference between the shallow and deep discs since their edge circle radii, r, only differ by 3·5%.

An example of the variation of the area A_p with disc angle at working depths of operation of 60, 120 and 180 mm is shown in *Fig. 8* for the shallow disc, mounted at a tilt angle of 20°.

4.1.2. *Vertical bearing area*

The area of soil contact with the rear convex spherical surface of the disc below soil level, projected on to a vertical plane normal to the direction of disc motion, is designated the vertical bearing area, A_b (area ABE in *Fig. 3*). It is zero when the disc angle is equal to or greater than the critical angle ($\beta > \beta_c$) which is the case for most disc angles used in practice. For values of $\beta < \beta_c$, the area will exist for depths between the critical depth, d_c, and the disc working depth.

The width, h, of the projected area at a depth, x, (*Fig. 3*) is given by

$$h = R_s'\{1 - \cos(\beta' - \beta)\} \qquad (7)$$

where $R_s' = R_o[1 - \{\sin(\alpha + \epsilon) - x/R_o\}^2]^{1/2}$ and β' is given by $\sin \beta' = \{x(2r \cos \alpha - x\}^{1/2}/R_s' \cos \alpha$.

The vertical bearing can be found by integrating the expression for h in Eqn (7) for values of x between d_c and d, so that

$$A_b = \int R_s'\{1 - \cos(\beta' - \beta)\}\, dx \qquad (8)$$

between the limits d_c and d, where values of the lower limit, d_c, are obtained from Eqn (5).

Values of the integral in Eqn (8) were obtained by numerical integration using a program employing Simpson's rule.

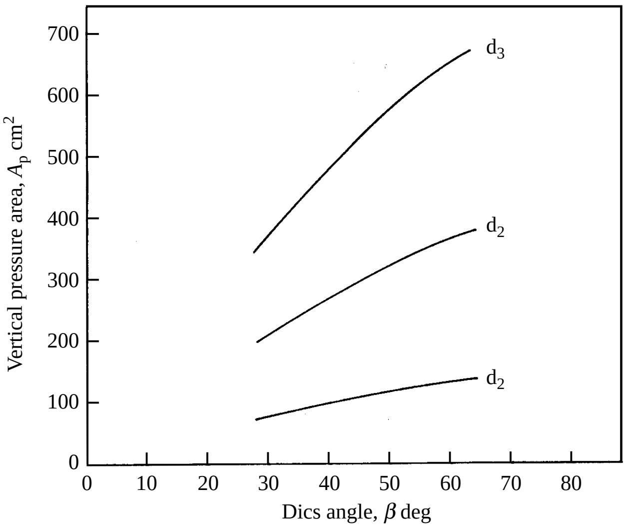

Fig. 8. Variation of vertical pressure area (A_p) with disc angle (β) for 51 mm disc concavity at a tilt angle α = 20° (working depths; d_1 = 60 mm; d_2 = 120 mm; d_3 = 180 mm)

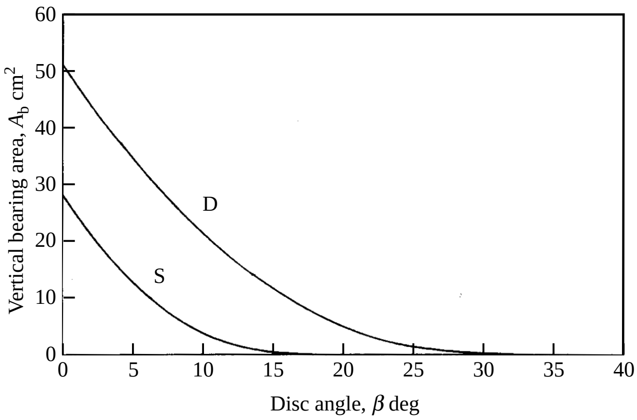

Fig. 9. Variation of vertical bearing area (A_b) with disc angle (β) at a working depth of 120 mm and a tilt angle α = 20° [disc concavity: 51 mm (S); 85 mm (D)]

Typical curves showing the variation of A_b with disc angle, for a 20° tilt angle and 120 mm working depth, are given in *Fig. 9* for both the shallow and the deep disc. These curves meet the disc angle axis for values of $\beta = \beta_c$, the critical angle. For a given tilt angle, α, the bearing area of the deep disc is considerably greater than that of the shallow disc, because of the much smaller radius of curvature of its spherical surface. The bearing area shows a marked increase with changes in the working depth of the disc, by a factor of four, over the range 180 to 300 mm.

4.2. *On a horizontal plane*

4.2.1. *Horizontal pressure area*

The area of the inner concave spherical surface of the disc which is engaged with the soil below soil level, when projected on to a horizontal plane, is designated the horizontal pressure area A_{ph} (area AFBCD in *Fig. 2*). For disc angles $\beta \geqslant \beta_c$, it can be shown that

$$A_{ph} = 0{\cdot}5R_s^2(2\beta_c - \sin 2\beta_c) + A_c \qquad (9)$$

where $A_c = 0{\cdot}5r^2(2\tau - \sin 2\tau)\sin\alpha$ (area ACD in *Fig. 2*) and $\sin\tau = L/r$ and L and R_s are given by Eqns (2) and (3), respectively.

If, however, the angle $\beta < \beta_c$, the projected area is given by

$$A_{ph} = 0{\cdot}5R_s^2\{2\beta - \sin 2\beta_c + \sin(\beta_c - \beta)\} + A_c \qquad (10)$$

In Eqns (9) and (10), β_c and R_s are given by Eqns (1) and (3), respectively. The angles β and β_c must be expressed in radians.

An example of the variation of the horizontal pressure area with disc angle is shown in *Fig. 10* for a 20° tilt angle and 120 mm working depth. When the disc angle $\beta > \beta_c$, the total area is constant as given by Eqn (9), since both A_{ph} and A_c are constant. For $\beta < \beta_c$, the area A_{ph} increases, although the area A_c is constant for all disc angles. For the practical range of β, the critical angle is generally exceeded, so the horizontal pressure area will be constant.

The tilt angle has a marked effect on A_{ph}, showing an increase of 79% as α increases from 15° to 25° at a working depth of 120 mm. Increasing the working depth from 60 to 180 mm results in an increase in this area by a factor of three for $\alpha = 20°$.

4.2.2. *Horizontal bearing area*

The area of the convex surface of the disc in contact with soil, below soil level, projected on to a horizontal plane is designated the horizontal bearing area A_{bh} (area AFB in *Fig. 2*). This area can also be derived from the projection of the disc on to a horizontal plane and is given by

$$A_{bh} = 0{\cdot}5R_s^2\{2(\beta_c - \beta) - 0{\cdot}5\sin(\beta_c - \beta)\} \qquad (11)$$

As for Eqns (9) and (10), the terms β_c and R_s are given by Eqns (1) and (3), respectively. The area $A_{bh} = 0$ when $\beta \geqslant \beta_c$. The angle $\beta_c - \beta$ must be in radians.

Fig. 11 shows the area for a 20° tilt angle and 120 mm working depth plotted against disc angle. Its general form can be seen to be similar to that for A_b, except that its magnitude is greater. The area shows a rapid reduction with increasing disc angle and is equal to zero when the disc angle is equal to or greater than the critical angle appropriate to the working depth. In general, however the angle β is greater than β_c in practice, so that the value of A_{bh} will be zero.

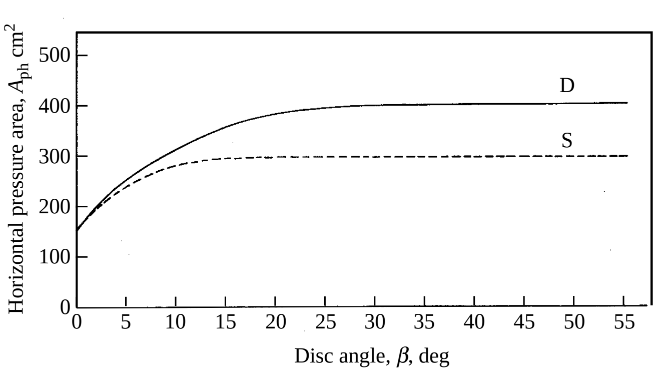

Fig. 10. Variation of horizontal pressure area (A_{ph}) with disc angle (β) at a working depth of 120 mm and a tilt angle $\alpha = 20°$ [disc concavity: 51 mm (S); 85 mm (D)]

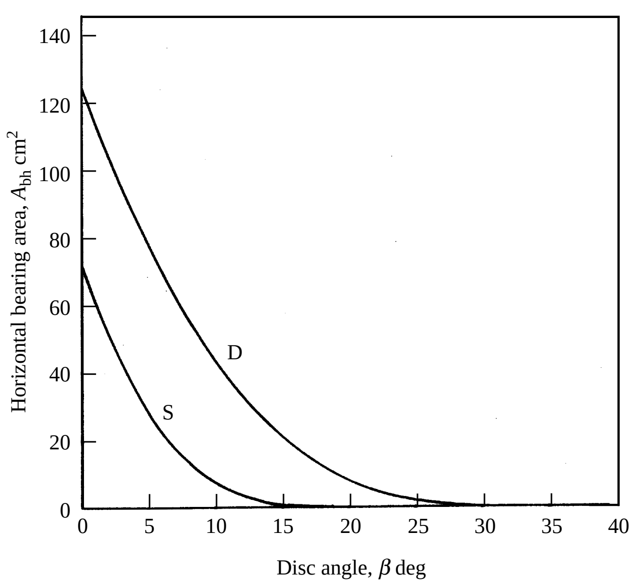

Fig. 11. Variation of horizontal bearing area (A_{bh}) with disc angle (β) at a working depth of 120 mm and a tilt angle $\alpha = 20°$ [disc concavity: 51 mm (S); 85 mm (D)]

5. Disc overlap and gang areas

5.1. *Overlap area* (A_{ov})

The area by which a preceding disc overlaps the area of the following disc, below soil level (area BFG in *Fig. 4*) is designated the overlap area, A_{ov}. It is the area by which the vertical pressure area of the following disc is reduced by the overlapping vertical pressure area of the preceding disc. This is shown in *Fig. 4* where the vertical pressure area of the following disc (DEFG) is overlapped by the vertical pressure area of the preceding disc (ABFC) by the area BFG, which is the area A_{ov}.

For a particular spacing, *g,* between the disc centres there is a critical disc angle, β_0, at which overlap between the cut soil areas will commence for a given working depth (d) and tilt angle (α). The critical angle is given by the expression

$$\sin\beta_0 = g\cos\alpha/2\{d(2r\cos\alpha - d)\}^{1/2} \quad (12)$$

Conversely, at a given disc angle (β), there is a critical disc spacing, g_c, at which overlap will begin, which is given by

$$g_c = 2\{d(2r\cos\alpha - d)\}^{1/2}\sin\beta\sec\alpha \quad (13)$$

The overlap area can be obtained by integration from the projections of the soil cut areas on to a vertical plane. It is given by the expression

$$A_{ov} = pq\sin^{-1}(1 - g^2/4q^2)^{1/2} + g(p - d) - pq\sin^{-1}(1 - d/p) - 0{\cdot}5pq(1 - g^2/4q^2)^{1/2} - q(1 - d/p)\{d(2p - d)\}^{1/2} \quad (14)$$

where $p = r\cos\alpha$ and $q = r\sin\beta$.

5.2. *Gang area* (A_g)

The area of soil cut by an individual disc, A_g, within a gang of discs, *Fig. 4,* is equal to the vertical pressure area of the disc (DEFG in *Fig. 4*) minus the overlap area (DEG in *Fig. 4*), that is

$$A_g = A_p - A_{ov} \quad (15)$$

The value of A_p is obtained from Eqn (6) and A_{ov} is given in Eqn (14) above.

The overlap angle, β_0, shows a small decrease with increasing tilt angle for a given disc spacing, but an increase with disc spacing, as expected. Over the practical range of values of β, there is overlap of the discs for practical values of α and g. The only exception is for a combination of the lowest value of the range for β (35°) and a high value of g (30 mm). For the shallow disc the critical angle, β_c, is exceeded for the practical range of g when there is overlap, but for the deep disc the value of g should be 224 mm or more to exceed β_c for the overlap condition.

For the practical range of g, there is generally disc overlap for typical ranges of α and β. Increasing the tilt angle results in only a small increase in critical spacing, g_c.

Over the practical range of β, the gang area A_g increases at a relatively slow rate as shown in *Fig. 12* for a 20° tilt angle, a 240 mm disc spacing and a 120 mm working depth. The overlap area reduces the vertical pressure area by 16% for a disc angle of 45° for the shallow disc conditions given in *Fig. 12.*

As the disc spacing is decreased below the critical spacing g_c for particular values of β and α, the gang area decreases as the overlap area increases.

5.3. *Uncut area* (A_u)

The ridge of soil which remains uncut,[14] which has a degree of asymmetry because of the tilt angle α, has an area, A_u, in a vertical plane (area CEF in *Fig. 4*) which is designated the uncut area. It lies between the two lowest points of adjacent discs (C and E in *Fig. 4*) which are at the maximum working depth (*Fig. 4*). The height of the cusp (F in *Fig. 4*) of this area, δ, above the lowest horizon worked by the discs (CE in *Fig. 4*) is given by the expression

$$\delta = r[1 - \{1 - g^2/4r^2\sin^2\beta\}^{1/2}]\cos\alpha \quad (13)$$

The area of uncut soil in a vertical plane is given by

$$A_u = gd - A_g \quad (14)$$

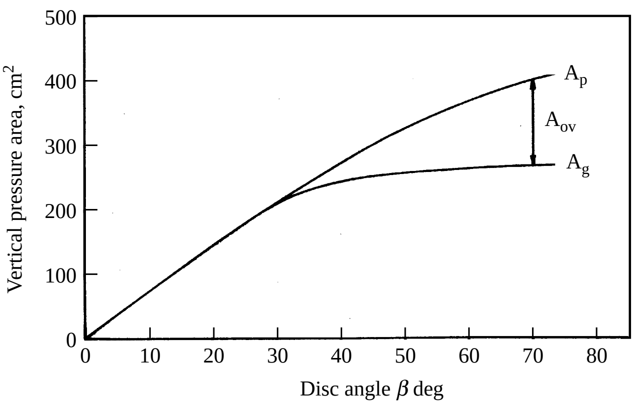

Fig. 12. Variation of vertical pressure areas (A_p, A_g and A_{ov}) with disc angle (β) for ganged discs at a working depth of 120 mm and a tilt angle α = 20° (disc concavity = 51 mm and disc spacing = 240 mm) (A_p: vertical pressure area; A_{ov}: overlap area; A_g: area cut by an individual disc)

where A_g is obtained from Eqns (6), (14) and (15).

Fig. 13 shows an example of the effect of disc spacing on the value of A_u/gd, expressed as a percentage, for the shallow disc. This represents the percentage of the cross-section of the area of the ridge of uncut soil between adjacent discs in relation to the nominal cut area which is the object of the cultivation process. The figure shows that at a typical 240 mm disc spacing, 12·5% of soil is uncut.

6. Disc sharpening

6.1. *Inside and outside sharpening*

A plough disc may be sharpened with the bevelled angle on the peripheral edge either internally or externally *Fig. 14*. The formulae developed in previous sections of the paper are applicable to a disc with internal sharpening. If the disc is externally sharpened the formulae need some modification for precise calculations.

For calculations which involve the rear spherical surface of the disc, i.e. the critical angle and depth (β_c and d_c) and the vertical and horizontal bearing areas (A_b and A_{bh}), it is necessary to modify some of the terms in the appropriate equations since the rear surface is effectively at a greater depth than the inner surface (*Fig. 14*).

The modified disc radius to be used, r_1, is given by

$$r_1 = r/\{1 + (2R_o - t)t/r^2\}^{1/2} \quad (18)$$

where r is the radius of the circumferential edge of the disc.

In addition modified terms for depth d and angle ϵ are required as given below

$$d_1 = d + (r - r_1)\cos\alpha \quad (19)$$

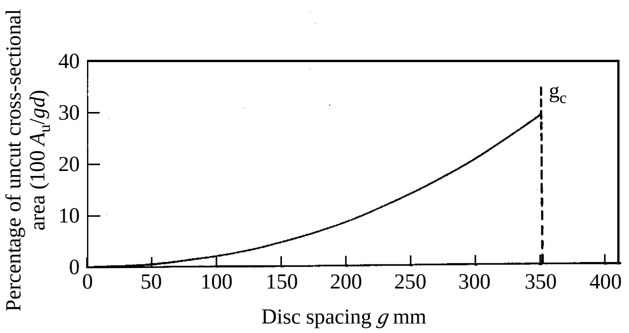

Fig. 13. Variation of the percentage of uncut soil cross-sectional area between adjacent discs in a gang with disc spacing (g) for a 51 mm disc concavity (working depth = 120 mm; tilt angle α = 20°; disc angle β = 45; critical spacing g_c = 35·1 mm)

and

$$\epsilon_1 = \sin^{-1}(r_1/R_o) \quad (20)$$

These terms are appropriate for Eqns (1), (2), (3), (5), (7), (8) and (11).

For calculations which concern the front spherical surface of the disc, i.e. the vertical and horizontal pressure areas, the unmodified values of r and d are employed but R_o is replaced by R_i. The value of ϵ to be used, however, is given by

$$\epsilon_2 = \sin^{-1}(r/R_i) \quad (21)$$

These values apply to Eqns (1), (2), (3), (6), (7) and (10).

In general, however, the differences between the calculated critical angles, depths and areas are relatively small because the difference in spherical radii of the discs is small. For the shallow disc the difference is 0·55% and for the deep disc it is 0·9%. The difference in disc radii is also small at 5% and 3% for the shallow and deep discs, respectively. Overall, the differences found between the inside and outside sharpening cases do not change the overall conclusions concerning the effects of the critical disc parameters.

6.2. *Effect of outside sharpening on critical disc angle*

The flat bevelled surface formed by external sharpening (*Fig. 15*) may be at an angle greater than the critical angle appropriate to the outer spherical surface at soil level. The angle made by the bevelled surface in the horizontal plane to the direction of motion of the disc, at soil level, is given by

$$\psi_s = \pi/2 - \tan^{-1}[\{[d(2r\cos\alpha - d)]^{1/2} - [d'(2r'\cos\alpha - d')]^{1/2}\}/w\cos\psi] \quad (22)$$

where $r' = r - w\sin\psi$ and $d' = d - w\sin(\psi - \alpha)$.

The term, w, is the width of the bevelled edge in a diametral plane of the disc spherical surfaces and is given by

$$w = R_i\{(R_o^2/R_i^2 - \cos^2\delta)^{1/2} - \sin\theta\} \quad (23)$$

The angle, θ, is the angle between the bevelled surface and the tangent to the inner spherical surface of the disc in a diametral plane. The angle, ψ, is the angle between the bevelled surface and a line normal to the disc plane in the diametral plane, given by

$$\psi = \pi/2 - \theta - \epsilon \quad (24)$$

For external sharpening, the angle ψ must be greater than the tilt angle α to give clearance to the bevelled surface at the lowest point of the disc in the soil. In

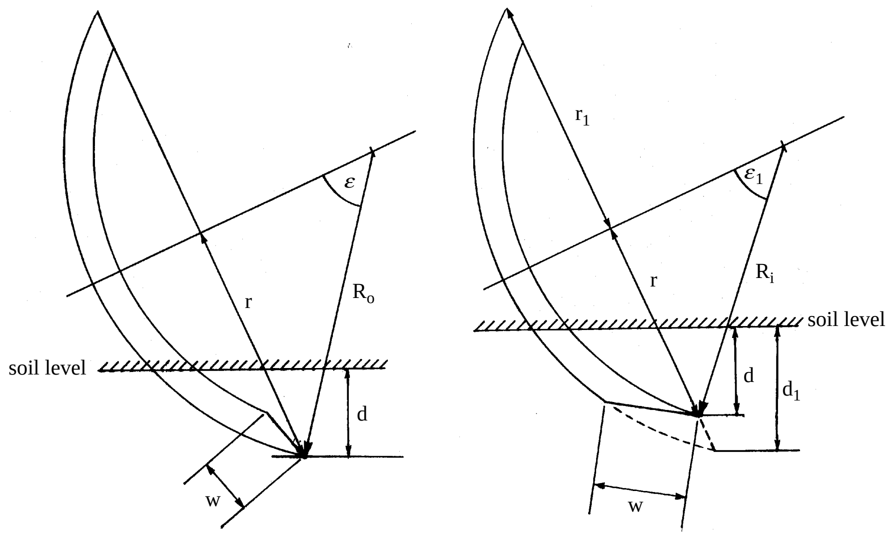

Fig. 14. Geometrical parameters for inside and outside sharpened discs

practice, this means that ψ should be greater than values of α in the range 15° to 25°.

For practical values of ψ, say in the range 15° to 30°, the critical angle in the plane of the soil surface for the flat bevelled surface is considerably larger than the critical angle for the rear spherical surface of the disc. This means that the bevelled surface may be in contact with soil for combinations of tilt and disc angle used in practice. There was little difference between the critical angles of the bevelled surface for the shallow and deep discs.

The contact of the bevelled surface is applicable over much of the working depth. For example, for a tilt angle of 20° and a disc angle of 45°, there is contact down to within 11·3 mm of the disc lowest point at a working depth of 120 mm.

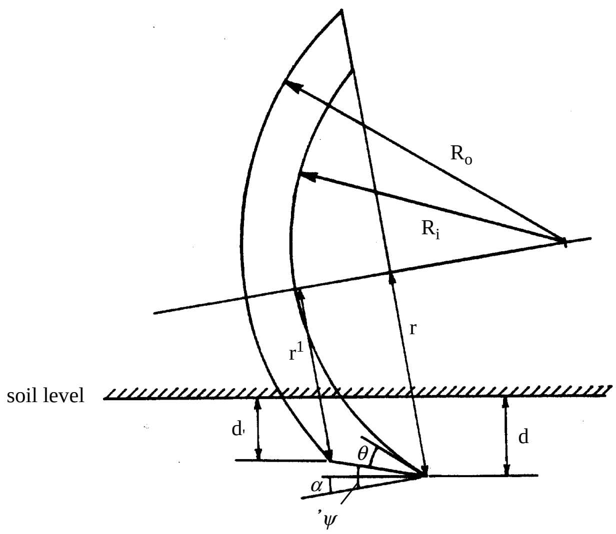

Fig. 15. Geometrical parameters and clearance angles for an outside sharpened disc

7. Discussion

The calculated values of the critical angle β_c show that for the range of disc angles used in practice (35° to 55°) the critical disc angle will be exceeded for both the shallow and deep discs over the working range of depth in the soil.

The values of critical angle are greater for the deep disc, as would be expected. The effect of the tilt angle α on critical angle is relatively small over the practical range (15° to 25°). The consequence of the critical angle being exceeded is that the bearing area of the disc will be zero in practice, so that there is no scuffing on the rear spherical surface of the disc and only the pressure area on the front spherical surface is significant.

The disc tilt angle, α, had a relatively small influence on the magnitude of the vertical pressure area A_p which was similar for the shallow and deep discs, because their circumferential diameters were nearly equal. Increasing disc angle and working depth, significantly increased the values of A_p.

The horizontal pressure area A_{ph} was constant for practical values of disc angle greater than the critical angle. The area A_{ph} showed significant increases for increasing values of tilt angle and soil working depth. The deep disc gave greater values of the area than the shallow disc.

The variation of the vertical and horizontal bearing areas and the critical depth is presented in Sections 4.1.2, 4.2.2 and 3.2 for values of disc angle up to the critical angle β_c. The data is of interest for cultivating discs set at angles less than the critical angles.

The disc angle (β) in the practical range will result in disc overlap at a typical soil working depth and for the practical range of disc spacings (180 to 300 mm). The tilt angle (α) has only a small effect on the angle β_0 at which disc overlap occurs. For the deep disc, the spacing of discs should be 240 mm or more in order to exceed the critical angle β_c. The range of practical disc spacings is less than the critical spacing at which disc overlap occurs.

The soil area cut by an individual disc, A_g, increases relatively little (12·5%) over the practical range of disc angles because of the reduction resulting from the overlap area A_{ov} over the range. The effect of overlap area also results in a reduction in the cut area at an increasing rate as disc spacing is reduced. The area of cross-section of uncut soil between adjacent discs is, typically, between 7% and 20% of the nominal cut area, over a range of disc spacing from 180 to 300 mm.

The results from the formulae developed in this paper were compared with limited results presented by other workers. McGreary and Nichols[1] present results for critical disc angle and vertical pressure and bearing areas. Their results were determined graphically and show good agreement with calculated values from appropriate expressions presented in this paper. Abo el Ees[7] and Abo El Ees and Wills[8] give results from formulae developed for a disc with zero tilt angle ($\alpha = 0°$). Their results for critical angle and vertical and horizontal bearing areas are in agreement with values calculated from the derivations of this paper when $\alpha = 0°$.

8. Conclusions

For the range of disc angles used in practice (35° to 55°) it was found that the critical angle was exceeded, so that the bearing area was zero and there was no scuffing on the rear spherical surface. This was true for both the shallow and deep discs and for the practical range of tilt angles (15° to 25°).

The vertical pressure area increased with both disc angle and working depth, but was only slightly affected by tilt angle. There was little difference in this area for the shallow and deep discs.

The horizontal pressure area increased with both tilt angle and depth, but was not affected by disc angle when it was greater than the critical angle. The deep disc had larger values of the area than the shallow disc.

When discs are working in a gang, the practical ranges of disc angle and spacing are such that overlap of the areas of soil cut will occur. The cultivated area of an individual disc increased relatively little over the practical range of disc angles, for a given spacing, because of the overlap effect. An area of uncut soil remains between adjacent discs in a gang.

The effect of inside and outside sharpening of discs was considered in relation to the geometrical analysis and it was shown that it had little effect on the overall conclusions because a disc is effectively a thin shell.

The effect of outside sharpening on the critical disc angle was examined. For practical values of the bevel angle, the bevelled flat surface will be in contact with soil for the range of tilt and disc angles used in practice, from the soil surface down to a high proportion of the depth of soil working. The bevel angle must be greater than the tilt angle to avoid scuffing at the lowest point of the disc within the soil.

References

[1] **McCreery W F; Nichols M L** The geometry of discs and soil relationships. Agricultural Engineering 1956, **37**(12): 808–812, 820

[2] **Harrison H P; Thivavarnvongs T** Soil reaction forces from laboratory measurements with discs. Canadian Journal of Agricultural Engineering 1956, **18**(1): 49–53

[3] **Reaves C A; Gill W R; Bailey; A C** The influence of width and depth of cut on disc forces. Transactions American Society of Agricultural Engineering 1981, **24**(3): 572–578

[4] **Gill W R; Reaves C A; Bailey A C** The influence of harrow disc curvature on forces. Transactions American Society of Agricultural Engineering 1981, **24**(3): 579–583

[5] **Gill W R; Bailey A C; Reaves C A** The influence of harrow disc curvature on soil penetration. Transactions American Society of Agricultural Engineering 1982, **25**(5): 1173–1180

[6] **Godwin R J; Seig D A T; Allot M** Soil failure and force prediction for soil engaging discs. Soil Use and Management 1987, **3**(3): 266–269

[7] **Abo El Ees N A E H** The use of curved discs as furrow openers on direct drills. Ph. D. Thesis, University of Newcastle upon Tyne, 1978

[8] **Abo El Ees N A E H; Wills B M D** An analysis of the geometric and soil working parameters of a curved vertical disc. Journal of Agricultural Engineering Research 1986, **35:** 277–286

[9] **Al-Ghazal A A** An investigation into the mechanics of agricultural discs. Ph. D. Thesis, Silsoe College, Cranfield Institute of Technology, 1989

[10] **Smith H P** Farm machinery and equipment. McGraw Hill, 5th Ed., 1964

[11] **Ingersoll R C** The development of the disk plough. Agricultural Engineering 1926, **7**(5): 172

[12] **Kepner R A; Barger R; Bainer E L** Principles of farm machinery. AVI Publishing Company, 3rd Ed., 1978
[13] **Gill W R; Reaves C A; Bailey A C** The effect of geometric parameters on disc forces. Transactions American Society of Agricultural Engineering 1980, **23**(2): 266–269, 27
[14] **Gill W R; Hendrick** The irregularity of soil disturbance depth by circular and rotating tillage tools. Transactions of American Society of Agricultural Engineering 1976 **19**(2): 230–233