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DEVELOPMENT OF THE ANALYSIS MODEL FOR WALL COUPLED BY FRAME STRUCTURES USING FE-BEM

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Abstracts

In this study, methodologies of the finite element and boundary element are combined to achieve an efficient and accurate analysis of frame structure containing shear wall. Our model analyzes the frame by employing the finite element method and the shear wall by boundary element method. This study is applicable to a specific situation, where the boundary element is surrounded by finite elements. If material properties of the shear wall are a lot smaller than its frame structure, the displacement shape of each node can be calculated to a great accuracy. However, it is worthy to note that the displacement shape is approximated more accurately when the solution of displacement is the larger.

Keyword: FEM, BEM, Coupling, Shear Wall, Stiffness Matrix, Linear Element

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ABSTRACT

In this study, methodologies of the finite element and boundary element are combined to achieve an efficient and accurate analysis of frame structure containing shear wall. Our model analyzes the frame by employing the finite element method and the shear wall by boundary element method. This study is applicable to a specific situation, where the boundary element is surrounded by finite elements. If material properties of the shear wall are a lot smaller than its frame structure, the displacement shape of each node can be calculated to a great accuracy. However, it is worthy to note that the displacement shape is approximated more accurately when the solution of displacement is the larger.

INTRODUCTION

Korea has small region and high population density. Agricultural engineering of Korea have studied structural analysis and design related to facility design because of using engineering technique for rural environment preservation and agricultural production increase.

Agricultural facility, of which frame rigidity is small relatively, is used for various purposes and it is hard to obtain reliable result for agricultural facility design through once analysis. Therefore, by using merit of the finite element and boundary element method, which is major structural analysis method, there is objective of this paper to obtain analytical result with high accuracy.

The finite element method is the general numerical technique for frame structures and practical engineering problems, which contains various geometry elements. Furthermore, the method is quite useful

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when one wish to deal with problems of nonlinear material and geometry. Nevertheless, there still are several issues to be. For instance, in general, there exist situations when artificial boundary condition is necessary, or when all domains is needed to be meshed into the finite elements in order to obtain any displacement of structure.

Once the fundamental solution, which governs the domain equation, is found, the boundary element method only requires discrete of the surface instead of the volume. For this reason, boundary element codes are easier to use with already existing solid modelers and mesh generators.

For example, boundary element method is useful for solving a singularity problem that occurs at stress concentration and crack. It is also useful for other finite or infinite problems of foundation and sea. However, the boundary element method is more difficult to apply in problems, which involve complicated shapes and non-linearity. In such cases, the finite element method would be a more suitable tool.

In this study, by employing FE dominant method in which boundary stiffness matrix is transformed into finite element stiffness matrix, boundary element and finite element methods are combined to analyze frame structure with walls. Up to this day, most finite element and boundary element has been jointed with the partial interface boundary among the total boundary. Nevertheless, in this study, the boundary element method is surrounded by the finite element methods. Since the frame is analyzed through the finite element method while the wall is analyzed through the boundary element method, the frame with wall can be analyzed simultaneously.

Many numerical methods have been developed to solve engineering problem. But these numerical methods have problems when integral domain is different though boundary of domain meets boundary condition. These numerical methods for engineering problems are matrix form to express integral region as numerical solution. If these numerical methods have proper rule and combination between matrixes, the application of these matrixes can be enlarged from combining structure with structure to combining structure with fluid.

Coupling of the finite element method and the boundary element method

Formulation of finite element method

There are wall surrounded by stiffness frame as example to combine finite element method and boundary element method. The finite element method by shape function and the stiffness matrix method for frame structure analysis carry identical accuracies in a two-dimensional linear frame structure. However, due to the fact that stiffness matrix method consumes less repetition in computing, it is often preferred over the finite element method.

The equation (1) shows element stiffness matrix that is used over the finite element method.

$$\begin{bmatrix} \frac{AE}{L} & 0 & 0 & -\frac{AE}{L} & 0 & 0 \\ 0 & \frac{12EI}{L^3} & \frac{6EI}{L^2} & 0 & -\frac{12EI}{L^3} & \frac{6EI}{L^2} \\ 0 & \frac{6EI}{L^2} & \frac{4EI}{L} & 0 & -\frac{6EI}{L^2} & \frac{2EI}{L} \\ -\frac{AE}{L} & 0 & 0 & \frac{AE}{L} & 0 & 0 \\ 0 & -\frac{12EI}{L^3} & -\frac{6EI}{L^2} & 0 & \frac{12EI}{L^3} & -\frac{6EI}{L^2} \\ 0 & \frac{6EI}{L^2} & \frac{2EI}{L} & 0 & -\frac{6EI}{L^2} & \frac{4EI}{L} \end{bmatrix} \begin{Bmatrix} x_1 \\ y_1 \\ \theta_1 \\ x_2 \\ y_2 \\ \theta_2 \end{Bmatrix} = \begin{Bmatrix} f_{x_1} \\ f_{y_1} \\ m_1 \\ f_{x_2} \\ f_{y_2} \\ m_2 \end{Bmatrix} \qquad (1)$$

Where A : Section Area, E : Elastic Modulus, L : Length of Element,
I : Moment of Inertia, x : Displacement of x direction, y : Displacement of y direction,
θ : Rotation Angle, f : Nodal Load, m : Nodal Moment, $[k_{local}]$ = Stiffness Matrix.

The equation (2) is transform matrix for stiffness matrix of equation (1).

$$[T] = \begin{bmatrix} \cos\theta & -\sin\theta & 0 & 0 & 0 & 0 \\ \sin\theta & \cos\theta & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & \cos\theta & -\sin\theta & 0 \\ 0 & 0 & 0 & \sin\theta & \cos\theta & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \qquad (2)$$

The equation (3) is global stiffness matrix.

$$[T]\ [k_{local}]\ [T]^{-1} = [K_{global}] \qquad (3)$$

Therefore, total structure system is following.

$$[K]\ \{U\} = \{F\} \qquad (4)$$

Formulation of Boundary Element Method

Boundary Integration Equation

The boundary integration equation is obtained from virtual work theorem. After the boundary condition, the divergence theorem, and the fundamental solution have been applied to the integration equation step by step, the boundary integration equation can be shown as follows.

$$c_l(i)u_l(i) + \iint_S u_k p_{lk}^* dS = \iiint_V b_k u_{lk}^* dV + \iint_S p_k u_{lk}^* dS \qquad (5)$$

Where S : Surface, V : Volume, l : Direction (1 or 2),

$u_l(i)$: Displacement of i node, u_k : Displacement of k node,

b_k : Force of k node, p_k : Traction vector of k node,

$c_l(i)$: Coefficient which is decided by boundary shape of i node,

u_{lk}^* , p_{lk}^* : Surface displacement and traction vector to be operated on from k to l by unit load of node i .

Boundary Element Equation

The boundary element such as followings is supposed for two-dimensional homogeneous elasticity equation.

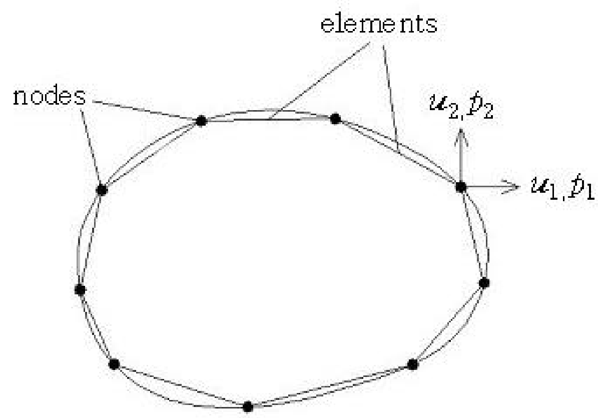

Fig. 1. Linear boundary elements

$$u_{lk}^* = u^* = [u_1 \quad u_2]^T, \quad p_{lk}^* = p^* = [p_1 \quad p_2]^T, \quad b_k = b = [b_1 \quad b_2]^T \qquad (6)$$

Therefore, equation (6) is as follows.

$$c(i)u(i) + \int_C p^* u ds = \iint_R u^* b dx_1 dx_2 + \int_C u^* p ds \qquad (7)$$

Where R : Domain region,

C : Boundary region.

If the equation (7) is discrete, the result is as follows.

$$c(i)u(i) + \sum_{j=1}^{N} \left\{ \int_{C_j} p^* ds \right\} u_j = \iint_R u^* b dx_1 dx_2 + \sum_{j=1}^{N} \left\{ \int_{C_j} u^* ds \right\} p_j \qquad (8)$$

Here, the body force term can be expressed as follows.

$$B_i = \iint_R u^* b dx_1 dx_2 = \sum_{s=1}^{M} \left\{ \sum_{k=1}^{l} (u^* b)_k w_k \right\} A_s \qquad (9)$$

Where M : Number of the interior cells, A : area of the interior cells, w_k : weighting value.

If the body force term is a negligible quantity and $\overline{H_{ij}}$ is defined as $\overline{H_{ij}} = \int_{C_j} p^* ds$ and $\overline{G_{ij}}$ as $\overline{G_{ij}} = \int_{C_j} u^* ds$, the equation (8) is can be displayed as follows.

$$c(i)u(i) + \sum_{j=1}^{N} \overline{H_{ij}} u_j = \sum_{j=1}^{N} \overline{G_{ij}} p_j \qquad (10)$$

Here, suppose H_{ij} as followings.

$$H_{ij} = \begin{cases} \overline{H_{ij}} & , \quad if \; i \neq j \\ \overline{H_{ij}} + c(i) & , \quad if \; i = j \end{cases} \qquad (11)$$

Therefore, the result is followings.

$$\sum_{j=1}^{N} H_{ij} u_j = \sum_{j=1}^{N} \overline{G_{ij}} p_j \qquad (12)$$

If the equation (11) is expressed in a matrix form, the result is can be seen as follows.

$$[H]\{U\} = [G]\{P\} \qquad (13)$$

The Coupling of Finite Element Method and Boundary Element Method

Relationship of Forces and Tractions

Considering the iso-parametric linear elements, the work done over an element by its nodal forces can be written as

$$W = (u_x)_1 (F_x)_1 + (u_x)_2 (F_x)_2 + (u_y)_1 (F_y)_1 + (u_y)_2 (F_y)_2 \qquad (14)$$

In terms of element displacements and tractions, this work can be expressed as

$$W = \int_{C_l} (u_x p_x + u_y p_y) dC \qquad (15)$$

Where C_l is the element length.

Using the intrinsic coordinate, z, and the Jacobian of transformation, J(z), and the shape functions, $N_c(z)$, to describe the displacements, this work can be written as

$$W(z) = \int_{-1}^{+1} [N(z)(u_x)_1 + N_2(z)(u_x)_2] p_x(z) J(z) dz + \int_{-1}^{+1} [N_1(z)(u_y)_1 + N_2(z)(u_y)_2)] p_y(z) J(z) dz \quad (16)$$

Where $N_1(z) = \frac{1}{2}(1 - z)$, $N_2(z) = \frac{1}{2}(1 + z)$, $J(z) = \sqrt{[\frac{dx(z)}{dz}]^2 + [\frac{dy(z)}{dz}]^2}$.

By comparing Equations (14) and (15), we can conveniently establish a relationship between nodal forces and nodal tractions as follows:

$$(F_x)_1 = \int_{-1}^{+1} N_1(z) [\sum_{n=1}^{2} N_n(z)(t_x)_n] J(z) dz \quad (17a)$$

$$(F_x)_2 = \int_{-1}^{+1} N_2(z) [\sum_{n=1}^{2} N_n(z)(t_x)_n] J(z) dz \quad (17b)$$

If equation (17) is expressed with shape number, the result is followings.

$$(F_x)_1 = \int_{-1}^{+1} N_1(z) [N_1(z)(t_x)_1 + N_2(z)(t_x)_2] J(z) dz \quad (18a)$$

$$(F_x)_1 = \int_{-1}^{+1} N_1(z) N_1(z)(t_x)_1 J(z) dz + \int_{-1}^{+1} N_1(z) N_2(z)(t_x)_2 J(z) dz \quad (18b)$$

$$(F_x)_1 = [\int_{-1}^{+1} N_1(z) N_1(z) J(z) dz](t_x)_1 + [\int_{-1}^{+1} N_1(z) N_2(z) J(z) dz](t_x)_2 \quad (18c)$$

If the equation (18c) is expressed in a matrix form, the result is can be seen as follows.

$$(F_x)_1 = \begin{bmatrix} \int_{-1}^{+1} N_1(z) N(z) J(z) dz \\ \int_{-1}^{+1} N_1(z) N_2(z) J(z) dz \end{bmatrix}^T \begin{bmatrix} (t_x)_1 \\ (t_x)_2 \end{bmatrix} \quad (19)$$

From equation (19), the row and column elements of transformation matrix (M) are followings.

$$M_{11} = \int_{-1}^{+1} N_1(z) N_1(z) J(z) dz \quad (20)$$

$$M_{12} = \int_{-1}^{+1} N_1(z) N_2(z) J(z) dz \quad (20)$$

Also, $M_{21}\,and\,M_{22}$ can be produced by same methods.

If the equation (20) is expressed in a matrix form, the result is can be seen as follows.

$$\{F\}=[M]\{P\} \tag{21}$$

Combination of FEM and BEM

If equation (13) is multiplying by the inverse of matrix G, the result is the followings.

$$[G]^{-1}[H]\{U\}=\{P\} \tag{22}$$

If equation (21) is multiplying by the inverse of matrix M, the result is the followings.

$$[M]^{-1}\{F\}=\{P\} \tag{23}$$

By combining equation (22) and (23), the stiffness matrix for BEM can be seen as follows.

$$[M][G]^{-1}[H]\{U\}=\{F\} \tag{24}$$

$$[K]^{BEM}\{U\}=\{F\} \tag{25}$$

From equation (4), the stiffness matrix for FEM can be seen as follows.

$$[K]^{FEM}\{U\}=\{F\} \tag{26}$$

If equation (25) and (26) is expressed in total matrix form, the combination result is followings.

$$\begin{bmatrix} [K]^{(FE)} & 0 & 0 \\ 0 & [K]^{(BE)} & 0 \\ 0 & 0 & [K]^{(FE+BE)} \end{bmatrix} \begin{bmatrix} U^{(FE)} \\ U^{(BE)} \\ U^{(FE+BE)} \end{bmatrix} = \begin{bmatrix} F^{(FE)} \\ F^{(BE)} \\ F^{(FE+BE)} \end{bmatrix} \tag{27}$$

The material parts, which are combined with boundary and finite element method together in equation (27), are followings.

$$[K]^{(FE+BE)}=a[K]^{(FE)}+b[K]^{(BE)} \tag{28}$$

Examples of Applications

Modeling

Frame Materials are followings.

$$E = 2.1\times10^{6}\,kgf/cm^{2} = 2{,}100{,}000\,kgf/cm^{2}\,,\quad h = \frac{10\,m}{21.5} = \frac{1{,}000\,cm}{21.5} = 46.51\,cm \approx 50\,cm\,,$$

$$I = \frac{bh^{3}}{12} = \frac{25\times50^{3}}{12} = 260{,}416.67\,cm^{4},\quad b = 25\,cm,\quad A = 1{,}250\,cm^{2}$$

Minimum thickness of one-way beam (when using steel of $\sigma_y = 2{,}400\,kgf/cm^{2}$):

$$\frac{L}{21.5}(KOREA)\ \ ver\ \ \frac{L}{16}(ACI)$$

Shear Wall Materials are followings.

$$E = 2.1\times10^{5}\,kgf/cm^{2} = 210{,}000\,kgf/cm^{2},\ \ \nu = 0.1,\ \ L(Length\ \ per\ \ Frame) = 250\,cm$$

Boundary conditions of the applied model are shown in below.

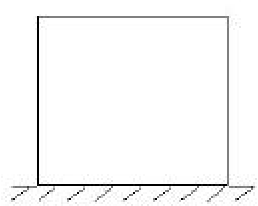

Fig. 2. Constraints of structure

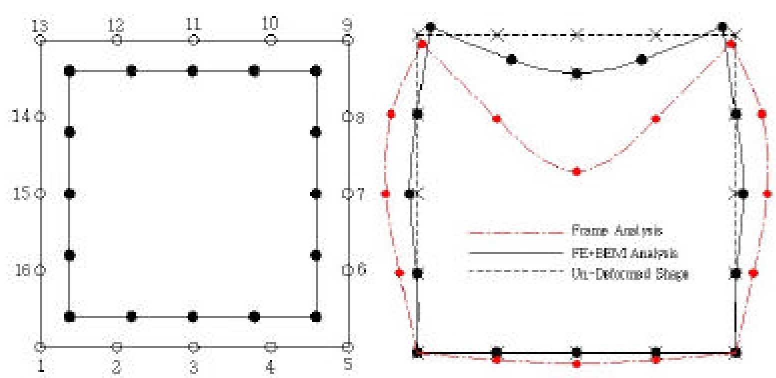

Fig. 3. The state in which the structure has been meshed into elements

To comparing differences between FE+BEM and exact solution, the lower modeling method using only frame is considered in calculating exact solution.

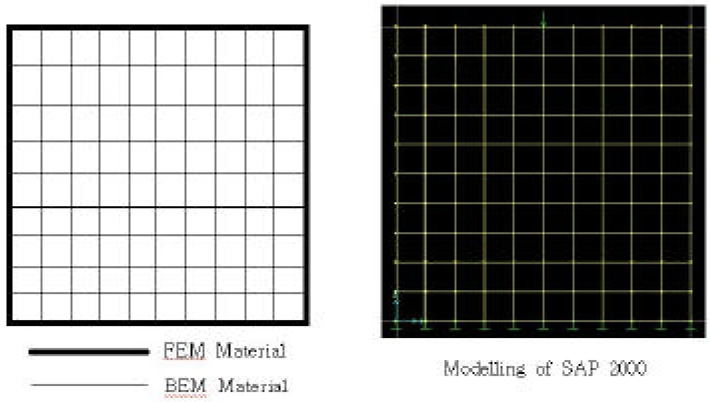

Fig. 4. The Modeling Method of SAP2000

Result

When the results from FEM(Frame alone without wall) and FE-BEM are $\alpha=1/2$ and $\beta=1/2$, the difference was about 20%. Those results are shown below.

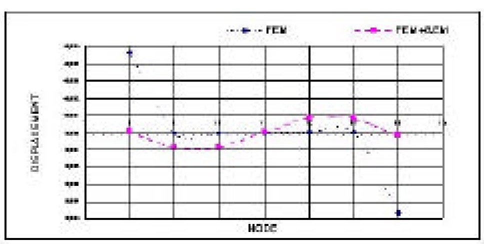

Fig. 5. Horizontal Displacement

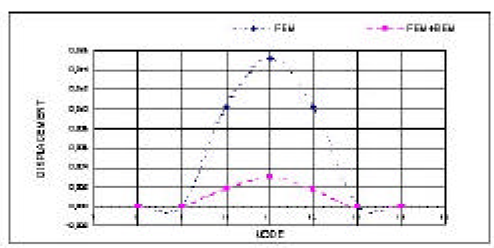

Fig. 6. Vertical Displacement

In above graph, node was chosen in which the largest displacement occurs. And then, the analysis result from FEM was compared with the result obtained from combining FEM and BEM. The result, which is obtained from the combined methods, shows less displacement than the result obtained from using Frame alone (vertical displacement: within 20%). Furthermore, since the point load is evenly distributed through wall, the displacement shows a slow distribution in overall.

Discussion

When one compares the elements from the FEM matrix with the elements from BEM matrix by applying identical material coefficient, it can be noticed that elements from the FEM matrix carry larger values. While combining the FEM and the BEM, at the shared boundary conditions, it may be inappropriate to employ simply averaged material value because influence from the BEM will dominate.

Therefore, following restrictions are proposed.

First, material value of wall should be much smaller than the material value of the Frame. In this case, since the solution is found based on the material value of Frame, more realistic solution can be obtained.

Second, the analysis result from combining FEM and BEM should be applied to the parts, which carry relatively large displacement. In other words, at the parts with large displacement, the displacement curve of Frame is in suitable agreement with its shape. Nevertheless, for the parts that carry small displacement, a minor deviation occurs, which may lead us to believe that the analysis result is unreliable.

Third, the combined analysis results are used to examine the overall behavior of the structure, which may change. Furthermore, it can be employed to locate the relative positions of the nodes.

New matrix equation to be used to combine FEM with BEM can be seen as below.

$$[K]^{(FE+BE)} = \frac{1}{\boldsymbol{a}}([K]^{(FE)} + \boldsymbol{b}[K]^{(BE)}) \qquad (29)$$

Where, $\boldsymbol{a}$: Approximately 2, b : Between 0 and 1

However, furthermore studies may be necessary to determine more accurate values for alpha and beta. It is believed that alpha and beta are closely related to Elastic Modulus and Poisson’s Ratio. Additional experiments should be fruitful to decide whether the analysis results are correct or not.

In current study, the combination was achieved by selecting the linear elements that have same number of node for each element. It is believed that such combination is feasible for constant element of BEM and linear element of Frame.

Conclusion

Analysis model was attempted for the frame structure, which carries wall, by applying FEM for the frame and BEM for wall, so that the finite element would be placed outside the analysis region of boundary element.

1. FEM and BEM has been formulated into a single equation.
2. As for the analysis results from combining FEM and BEM, the displacement curve and shapes were in good agreement at the parts that have large displacement.
3. New matrix equation has been formulated for the case of combining FEM with BEM.

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