COEFFICIENTS FOR QUANTIFYING SUBSURFACE DRAINAGE RATES

R. Wayne Skaggs

ABSTRACT. It is proposed that technical papers on drainage research studies and engineered design projects should report standard coefficients or parameters that characterize the hydraulics of the system. The following coefficients define key subsurface drainage rates that can be used to quantify and compare the hydraulics of drainage systems across sites, soils and geographic locations. (1) The steady subsurface drainage rate (cm/d) corresponding to a saturated profile with a ponded surface. This subsurface drainage rate defines the length of time that water remains ponded on the soil surface following large rainfall events. It is proposed that this rate be called the Kirkham Coefficient (KC) in honor of Professor Don Kirkham who derived analytical solutions for saturated drained profiles for most soil and boundary conditions of interest. (2) Drainage intensity (DI), which represents the drainage rate (cm/d) when the water table midway between parallel drains is coincident with the surface. The DI can be estimated by the Hooghoudt equation and is dependent on the effective saturated hydraulic conductivity of the profile, drain depth, spacing, and depth of the soil profile or restrictive layer. (3) The drainage coefficient (DC), which quantifies the hydraulic capacity of the system. This value is the rate (cm/d) that the outlet works can remove water from the site. It is dependent on the size, slope, and hydraulic roughness of the laterals, submains, mains, and, in cases where pumped outlets are used, the pumping capacity. Routine inclusion of these three coefficients in the documentation of research and design projects would facilitate comparison of results from different soils and drainage systems, and generally, the meta-analysis of data pertaining to drainage studies.

Keywords. Drainage, Drainage intensity, Drainage coefficient, Drainage nomenclature, Kirkham Coefficient.

Technical papers on drainage studies usually describe the drainage system design, site conditions, and soil properties in sufficient detail to enable the reader to quantify subsurface drainage rates for various water table positions. In most, but not all cases, this information is provided in the documentation of the study site. It is proposed herein that technical papers on drainage research studies and engineered design projects should report standard coefficients/parameters that characterize the hydraulics of the system. The coefficients described below define key subsurface drainage rates that quantify the hydraulics of a drainage system.

1. The steady subsurface drainage rate (cm/d) corresponding to a saturated profile with a shallow ponded surface. It is proposed that this coefficient be called the Kirkham Coefficient (KC) in honor of Professor Don Kirkham who derived analytical solutions for saturated flow to subsurface drains for a wide range of soils and boundary conditions.

2. Drainage intensity (DI) which represents the steady state drainage rate (cm/d) when the water table midway between parallel drains is coincident with the surface. The DI can be calculated by the Hooghoudt equation (Bouwer and van Schilfgaarde, 1963; Luthin, 1978) and is dependent on the effective saturated hydraulic conductivity of the profile, drain depth, spacing, effective radius of the drain, and equivalent depth to the restrictive layer.

3. Drainage coefficient (DC) which quantifies the hydraulic capacity of the system. This value is the rate (cm/d) that the outlet works can remove water from the site. It is dependent on the size, slope, and hydraulic roughness of the laterals, submains, mains, and in cases where the pumped outlets are used, the pumping capacity.

These coefficients quantify the hydraulics of a drainage system at a minimum level. Another parameter that would be useful is the time required for a standard water table drawdown. For example, the time required to lower the water table from the surface to a depth of 30 cm.

There is nothing new or novel about any of the above coefficients/parameters. The objectives of this article are to recognize the importance of a clear definition of the coefficients, to show by example how they interact to affect drainage rates and water table drawdown, and to emphasize the need for their inclusion in technical papers and research reports. For example, the effectiveness of Drainage Water Management (DWM) would logically increase with DC and DI, other factors being equal. The
inclusion of the above coefficients in research and design publications would make it possible to easily compare results in terms of the hydraulics of the system.

**QUANTIFYING THE SUBSURFACE DRAINAGE PROCESS**

The evolution of the water table resulting from drainage following a rainfall event that saturates the profile and ponds water on the surface is shown schematically in figure 1. The relationship between drainage flux, q, and water table elevation, m, midway between parallel subsurface drains, is shown in figure 1b. Drainage rates for specific water table positions 1-5 (fig. 1a) are denoted in figure 1b. Traditional drainage theory can be used to quantify the q(m) relationship of figure 1b. The so-called exact approach to describing the process is to solve the Richards equation for two-dimensional saturated and unsaturated flow subject to the appropriate boundary conditions (Nieber and Feddes, 1999). The equations and soil property inputs are nonlinear and numerical methods are required for solution. Early solutions for field-scale saturated and unsaturated flow to parallel drains were presented by Skaggs and Tang (1976). McWhorter and Marinelli (1999) point out that even this complex approach does not consider the dynamics of the air phase as drainage and subsequent infiltration progress. Simpler approaches approximate or ignore unsaturated flow and use the concept of specific yield, or drainable porosity, the Dupuit-Forchheimer assumptions and the Boussinesq equation to predict drainage rates and water table drawdown (Youngs, 1999). When the profile is saturated and water is ponded on the surface (position 1), the drainage rate may be calculated by equations developed by Kirkham (1957) (denoted by KC in fig. 1b). Flow lines are concentrated near the drain for this case with a large percentage of the drainage water entering the profile within a horizontal distance equal to the drain depth (i.e., ±b). The condition shown is for parallel subsurface drain tubing or tile. Professor Kirkham developed exact mathematical solutions for drain tubes, ditches, layered soils, and a wide range of boundary and soil conditions for relevant two-dimensional saturated problems. The subsurface drainage rate is at a maximum at KC for the ponded surface condition in most all cases; therefore, it is a key parameter for quantifying the hydraulics of a drainage system. After the depth of surface water recedes due to drainage and evaporation to depth $S_2$ (fig. 1a), water can no longer move across the surface to the vicinity of the drains, the water table near the drain is drawn down (position 2) and the Kirkham equations for saturated profiles are no longer valid, even though the water table midway between the drains may still be above the surface. As the water table falls from position 2, it remains at or slightly above the surface midway between the drains for some period of time, and the drainage rate falls from KC along the vertical line (at m=100 for the example in fig. 1b). Drainage rates continue to decline until the water table midway between the drains is just coincident with the surface (position 3) and attains an approximately elliptical shape. At this point the drainage rate can be estimated with the steady state Hooghoudt equation (Bouwer and van Schilfgaarde, 1963):

$$q = 4 K_c \frac{m (2d_e+m) b}{L^2}$$ (1)

where q is drainage rate (cm/h), m is midpoint water table elevation above the drain, $K_c$ is the equivalent lateral hydraulic conductivity of the profile (cm/h), $d_e$ is the equivalent depth from the drain to the restrictive layer (cm), and L is the drain spacing (cm) (fig. 1a). The drainage rate for position 3 when the midpoint water table is at the surface may be defined as the subsurface drainage intensity, DI, which may be written in terms of the variables in figure 1 as,

$$DI = 4 K_c b \frac{2d_e+b}{L^2}$$ (2)

The drainage intensity is often called the drainage coefficient (DC) and has been used as a basis for estimating drain spacing and depth in both humid and irrigated arid and semi-arid regions (Luthin, 1978). However, reserving the term drainage coefficient to define the hydraulic capacity of the drainage system (including potential infrastructure limitations), as will be discussed below, will result in a more consistent use of this term. Additionally, defining drainage intensity (DI) as the rate water can move through the soil profile to the drain when the water table midway between the drains is at the surface further defines the in-field portion of the system, including the effect of soil properties. DI is an extremely useful parameter that defines an important drainage process and is used in design and layout of subsurface drainage. The water table condition associated with the definition of DI (position 3 in fig. 1) may develop when the drainage rate is equal to steady rainfall or irrigation, or, as discussed above, when the process is transient and drawdown at the midpoint is.
just beginning. The drawdown process as the water table falls from position 3 to position 4, and finally to drain depth, is obviously not steady state, but in most cases, proceeds slowly, and the drainage rate can be estimated by the Hooghoudt equation (Tang and Skaggs 1977). Drainage intensity is a function of the drain spacing and depth and the hydraulic transmissivity of the profile.

The values predicted by the Kirkham, Hooghoudt, ellipse (Donnan, 1946), or several other equations, quantify the rate of water movement through the soil to the drains for given water table elevations. Often the drainage rate may be limited by the hydraulic capacity of the drainage network rather than by how rapidly water can move through the soil profile to the drains. For instance, assume that the mains and sub-mains for the drainage system of figure 1 have a maximum hydraulic capacity of 2.5 cm/day. Then the drainage rate would not exceed this rate regardless of the rate water could move through the profile. Again, reserving the term drainage coefficient, DC, to describe the maximum rate of the drainage system not only keeps with the spirit of this term, it allows the system to be further defined with more descriptive parameter information (i.e., DI). It is proposed here that the hydraulic capacity continue to be called the drainage coefficient, DC. This is consistent with nomenclature used in the past as the hydraulic capacity is nearly always called the drainage coefficient, DC. Luthin (1978) referenced the Yarnell-Woodward charts that were used for decades to size collector drains in terms of the DC. The DC depends on the size of the area being drained and the parameters defining the outlet works, such as the diameter, slope and hydraulic roughness of the drains (mains, submains, and laterals), which may involve a network of pipes, ditches, and natural streams. In the case of pumped outlets, the DC obviously depends on the pumping capacity. The drainage rate is limited to DC, regardless of the water table position. So, for example, when the profile is saturated and water is ponded on the surface such that it could theoretically drain through the soil at a rate of KC (3.3 cm/d in fig. 1b), the actual rate would be limited by the outlet capacity to DC=2.5 cm/d. In this example, DC=D<KC, but DC may be greater than KC or less than DI, depending on the capacity of the outlet.

Some readers may disagree with my suggestion to relegate the use of the term drainage coefficient (DC) to defining hydraulic capacity of the drainage network. DC has been used in so many ways—as the hydraulic capacity as suggested here, as the DI defined herein, as the DI when the midpoint water table is 30, or 40, or 50 cm deep, etc.—that its meaning has become unclear. Luthin (1978) defined DC as the volume of water removed from the field by the drainage system in a 24 h period. He and many others (e.g., USDA-SCS, 1973; USDA-NRCS, 2010) used the DC as DI defined herein in equations (such as the Hooghoudt, eq. 1) to determine drain spacing, and in the Manning’s equation to determine size of the collector drains to carry the required hydraulic load. In the design phase of a project DC and DI would logically be equal. However, they are used to characterize different processes. One (DI) to define the rate water moves through the soil to the drains, and the spacing and depth of drains necessary to do the job, and the other (DC) to define the hydraulic capacity that must be provided to move the water from the field to the drainage outlet. In practice the two coefficients are usually not exactly equal, and are often substantially different. For example the effective hydraulic conductivity of the soil profile may be greatly different than assumed in design, resulting in a DI that is either much greater or much less than the hydraulic capacity of the outlet. Both DI and DC are needed to characterize the drainage system and its performance. In my opinion, it is not particularly important what term we use, but it is important that we have a common definition of what the term means. It is proposed here to clearly define the DC as the hydraulic capacity. While the drainage intensity has often been referred to as the DC in the past (e.g., Willardson, 1982; USDA-NRCS, 2010), the hydraulic capacity has always been called the drainage coefficient, DC. My recommendation is that we continue to call it DC, and that we agree on this unambiguous definition.

**Examples**

The following examples are presented to demonstrate the interaction between KC, DI, and DC. Referring to figure 1a, the examples are for drainage to parallel drains in a homogeneous soil with a drainable porosity of 0.05 (5%). Unlike figure 1a, the soil profile in these examples is assumed to be uniform with hydraulic conductivity of 5 cm/h for all examples except for figure 7 where a range of K values was used to demonstrate effects of DI on water table drawdown. The drain depth is 100 cm in all examples and the equivalent depth to an impermeable horizon is 200 cm below the surface, or 100 cm below the drain. Except for one case, the drain spacing is 30 m in all examples. The one exception is a 48 m spacing used to illustrate effects of changing the DI (fig. 6).

The Kirkham Coefficient (KC) is always greater than DI and may often be greater than DC. Equations or numerical methods for calculating either KC or DI do not usually consider the hydraulic capacity, as quantified by the DC, so the drainage rates calculated may be greater than DC. In all cases when KC or DI is greater than DC, the actual drainage rate is limited to DC, except perhaps for the initial stage of an event when the drainage network is filling with water.

Relationships between drainage flux, q, and water table elevation above the drain at a point midway between drains, m, as predicted by the Hooghoudt equation, and by a numerical solution to the Boussinesq equation, are plotted in figure 2 for DC=4.8 cm/d (1.9 in./d). The nonlinear Boussinesq equation was solved by numerical methods described in an earlier article (Skaggs, 1973) with modifications to consider the limiting flux boundary condition (DC) at the drain. The DI calculated by the steady state Hooghoudt equation is 1.6 cm/d, so, for an initially saturated profile, it is assumed the drainage rate falls from DC=4.8 cm/d to DI=1.6 cm/d along a vertical line at m=100 cm as drainage proceeds. This is a calculation that doesn’t consider the transients and assumes the drainage rate at specified midpoint water table elevations may be approximated with the steady state
Hooghoudt equation. The solution to the Boussinesq equation used a boundary condition at the drains that limited the maximum flux at the drain to DC=4.8 cm/d. This solution considered the transients as the water table receded during drainage. The drainage rate predicted by this method was 2.2 cm/d for m=100 cm. That is, the DI determined from the transient Boussinesq equation is somewhat greater than predicted by the steady state Hooghoudt equation. Conversely, the Hooghoudt equation predicted slightly greater drainage rates than the Boussinesq for m values in the 30 to 90 cm range for this example. Overall, experience indicates that drainage rates to parallel drains can be reasonably well approximated with the Hooghoudt equation for most cases of interest. Notable exceptions include drainage during and shortly after rainfall or irrigation events (Skaggs et al., 2008).

The q(m) relationships for the same profile as considered in figure 2, but for DC values of 1.25 cm/d (0.5 in./d) and 0.63 cm/d (0.25 in./d) are plotted in figures 3a and 3b, respectively. Note that in both cases the maximum drainage rate is equal to DC and remains so until the water table falls to m values less than 80 cm for DC=1.25 cm/d and less than 50 cm for DC=0.63 cm/d. Note also that the q(m) relationships predicted by the Hooghoudt equation is close to that predicted by the Boussinesq equation throughout the drainage process for both cases.

When the drainage rate is limited by the DC, the rate the outlet works can remove water from the field, the water table near the drains rises, or backs up, reducing the hydraulic gradient near the drains. This is shown in figure 4 where water tables predicted by numerical solutions to the Boussinesq equation for DC=0.63 cm/d are compared to those for DC=4.8 cm/d at times when the midpoint water table was 50, 70, and 90 cm above the drain. Drainage and water table drawdown are obviously slower for DC=0.63 than for DC=4.8 cm/d (fig. 5) so the time when the midpoint water table recedes to a given elevation is greater for the smaller DC value. For example, 78 h were required for the midpoint water table elevation to fall from the surface to m=70 cm for DC=0.63 cm/d compared to only DC=0.63 cm/d (0.25 in./d) (broken curves) and 4.8 cm/d (2 in./d) (solid curves).
36 h for DC=4.8 cm/d. Note also that, when m=70 cm, the water table elevations at x=0 and x=L were about 33 cm above the drain for DC=0.63 cm/d, whereas the corresponding predicted water table for DC=4.8 cm/d intersected the drain. In general, when the drainage rates are limited by the hydraulic capacity (DC) rather than by the rate water can move through the soil to the drain, the water table backs up over the drains until the drainage rate falls below DC. A practical way of determining whether drainage rates are limited by DC or DI is to simply examine conditions during very wet periods. If water is backed up for relatively long periods in the mains and laterals, drainage is limited by DC.

The effect of the DC on water table drawdown is shown in figure 5 for the above examples. As indicated in figure 3, drainage rates may be limited by DC when water tables are close to the surface and this may include a substantial part of the drawdown event. Results in figure 5 show that the time required to lower the water table from the surface to a depth of 30 cm, for example, increases from 39 h for DC=2.5 cm/d (1 in./d) to 44 h for DC=1.6 cm/d (0.63 in./d) to 78 h for DC=0.63 cm/d (0.25 in./d). A low DC limits drainage rates during the time when they would otherwise be at their highest values, thereby extending the drainage process and resulting in wetter field conditions.

Results in figures 3-5 indicate the need for matching the DC and DI. This need is more clearly demonstrated by results in figure 6, where drawdown is predicted for four combinations of drainage intensity (DI=0.63 and 1.6 cm/d) and drainage coefficient (DC=0.63 and 1.6 cm/d). A DI of 0.63 cm/d was obtained by using a drain spacing of 48 m rather than 30 m. All other drainage system parameters were the same as used in the previous examples. Results in figure 6 show that a design with DC=DI=0.63 cm/d (0.25 in./d) would result in mid-pipe water table drawdown for an initially saturated profile of 30 cm in 112 h. Assume for example that the DC is constrained to 0.63 cm/d by an undersized main drain. Increasing the size of the main to provide a DC of 1.6 cm/d would reduce the time from 112 to 94 h. Alternatively, reducing the lateral drain spacing to 30 m such that DI=1.6 cm/d, while maintaining DC=0.63 cm/d, would result in drawdown of 30 cm in 77 h. While increasing DI is more effective than increasing DC in this case, the full effect of increasing the drainage intensity would not be realized without also increasing DC. When both DI and DC are increased to 1.6 cm/d, the time required for 30 cm midpoint water table drawdown drops substantially to 44 h.

The combined effects of DI and DC on midpoint water table elevation are shown in figure 7. These results were obtained by numerically solving the Boussinesq equation for parallel drains spaced 30 m apart and 1 m deep and the same soil properties assumed for the examples of figures 3-5, with one exception. The hydraulic conductivity, K, was varied over a range of 1.98 to 15.9 cm/h to give DI values in cm/d (in./d) of 0.63(0.25), 1.27(0.5), 1.6(0.63), 1.91(0.75), 2.54(1), 3.18(1.25), 3.81(1.5), and 5.08(2). The same DC values were assumed and the Boussinesq equation was solved to predict water table elevations for 64 combinations of DI and DC. Results in figure 7 show the importance of matching DC and DI in drainage system design. Ideally, drain depth and spacing should be determined to provide a DI that will

**Figure 5.** Effect of drainage coefficient (DC) on water table drawdown midway between parallel drains as predicted by numerical solutions to the Boussinesq equation for an initially saturated profile; KC=5 cm/d; DI=1.6 cm/d.

**Figure 6.** Effect of four combinations of drainage coefficient (DC) and drainage intensity (DI) on midpoint water table drawdown for an initially saturated profile (water table at surface) as predicted by numerical solutions to the Boussinesq equation. Referring to figure 1, solutions obtained for uniform soil with K=5 cm/h; drainable porosity, f=0.05; drain depth, b= 1 m, and d=1m; L=30 m for DI=1.6 cm/d; L=48 m for DI=0.63 cm/d.
optimize yields and profits. Results presented in the above examples show that the DC should be at least as large as the DI to avoid reducing drainage rates and effectiveness. In some cases DC is limited by drainage outlets on a watershed or larger scale. Constructing drainage systems with DI much larger than the DC would obviously not make economic sense in those cases.

Results in figure 7 may be used to make the important point that drainage systems should not be designed with DI/DC values greater than necessary. Simulation studies conducted for 10 eastern U.S. locations (Skaggs et al., 2006; Skaggs, 2007) showed that optimum DI depended on location, soil properties, and drain depth. Optimum DI for corn production at five Midwest U.S. locations varied from 0.8 to 1.2 cm/d, which is close to the 3/8 in/d (0.95 cm/d) criterion traditionally recommended in the region (Schwab et al., 1981). Drainage intensity greater than the optimum will lower the water table more than necessary, removing water that might otherwise be used by the growing crop. For example, if DI=DC=1.27 cm/d (0.5 in./d) the water table elevation 48 h after drainage began is 76.1 cm, or 23.9 cm below the surface (fig. 7). The drainable porosity is 5%, so 1.2 cm drained out of the profile in 48 h. Compare this to the results for DI=DC=5.08 cm/d (2 in/d) where the water table elevation after 48 h is 17.9 cm (fig. 7), or 82.1 cm deep. In this case, 4.1 cm would have drained out and there would have been 2.9 cm less water in the profile available to the crop than for the DI=DC=1.27 cm/d case. Not only does the greater than necessary DI cost more because more closely spaced drains are required, but the excess drainage may reduce both yields and profits. Further, greater than necessary DI and DC will increase losses of nitrate nitrogen to receiving waters exacerbating water quality problems (Skaggs et al., 2006).

**SUMMARY AND RECOMMENDATIONS**

The examples presented in this article show that the hydraulics of the drainage system, as represented by the Drainage Coefficient (DC) combine with the physics of flow of water through the soil to the drains, as reflected by the Kirkham Coefficient (KC) and drainage intensity (DI) to determine drainage and water table drawdown rates. The KC, DI, and DC coefficients suggested herein represent the minimum information needed to characterize a drainage site. More information, such as details on hydraulic conductivity by layer, would normally be given in research or design reports. The consistent inclusion of these standard coefficients (with perhaps others) is recommended in order to provide a common basis for comparison with other studies and designs. It is also recommended that the definition of drainage coefficient be standardized to refer to the hydraulic capacity of the drainage system, and that the terms Kirkham Coefficient and Drainage Intensity be used to quantify drainage rates as affected by the physics governing movement of water through the soil profile to the drain.
ACKNOWLEDGEMENT
The assistance of Dr. Lamyaa Negm, Research Associate, BAE, N.C. State University, with the 3-D graphics in this article is gratefully acknowledged.

REFERENCES